Laboratory 2
Design of an Instrumentation Amplifier

OBJECTIVE

To design and implement an op-amp based instrumentation amplifier.

BACKGROUND

An instrumentation amplifier is typically the first stage in an instrumentation system. It is used to amplify the signal produced by a transducer such as a thermocouple or a strain gauge. An instrumentation amplifier is a difference amplifier i.e., it amplifies the voltage difference between its two input terminals, neither of which is required to be a signal ground. An instrumentation amplifier should have the following characteristics: high input resistance, high voltage gain, and high common-mode-rejection-ratio (CMRR).

Common-mode signals and CMRR

A common-mode signal is a signal that is common to both input terminals of an amplifier. In an instrumentation system, the wires leading from the transducer to the amplifier can pick up common mode interference. The common-mode signal can be of the order of a few volts, while the difference signal to be amplified can be much smaller e.g., in the millivolt range. Instrumentation amplifiers must be able to amplify the desirable difference signal (denoted $v_d$) and reject the undesirable common-mode signal (denoted $v_{cm}$). The CMRR is a figure of merit of an amplifier that specifies how good it is at rejecting common mode signals. Let $A_d$ be the differential voltage gain of the amplifier and $A_{cm}$ be the common-mode voltage gain. The CMRR is defined as follows: $\text{CMRR} = |A_d| / |A_{cm}|$.

Consider an op-amp operating open-loop as depicted in Fig. 2-1. The difference signal is $v_d = v_p - v_n$, and the common-mode signal is $v_{cm} = (v_p + v_n)/2$. In the case of Fig. 2-1, the difference signal is $v_d = 0$, and the common mode signal is $v_{cm} = V$. If the op-amp is ideal, the output signal is $v_o = A_d v_d$ (only the difference signal appears at the output). The output signal is thus $v_o = 0$ if...
the op-amp is ideal. However the common mode signal does propagate to the output if the op-amp is non-ideal. For a non-ideal op-amp, the output signal is

\[ v_o = A_d v_d + A_{cm} v_{cm} \quad \text{Eq. 2-1} \]

The typical voltage gain \( A_d \) of an LM 741 op-amp is 200,000 and the typical CMRR is 90dB. Note that the CMRR in dB is just 20 \( \log \left( \frac{|A_d|}{|A_{cm}|} \right) \).

The configuration of a basic difference amplifier is depicted in Fig. 2-2. Using ideal op-amp analysis it is easy to show that the input resistance of the amplifier is \( 2R_1 \) and that the output signal of the amplifier is \( v_o = \frac{R_2}{R_1} \cdot (v_2 - v_1) \). The amplifier thus has zero common-mode gain and a differential voltage gain of \( \frac{R_2}{R_1} \). Note that the common-mode gain is not zero if the op-amp is non-ideal, or if the resistors labeled \( R_1 \) (or \( R_2 \)) do not have identical values.

Let \( v_p \) and \( v_n \) denote the potentials at the + and - terminals of the op-amp of Fig. 2-2, respectively. Let us now analyze the basic difference amplifier assuming that the op-amp is non-ideal. Specifically, let us assume that the op-amp has finite gain \( A_d \) (resulting in \( v_p - v_n \) being non-zero). We will also assume that the op-amp has infinite input resistance (no current flows into the + or - terminal).

Let \( v_d \) and \( v_{cm} \) denote the difference and common-mode signals of the difference amplifier, of Fig. 2-2, respectively. We have

\[ v_d = v_2 - v_1 \quad \text{and} \quad v_{cm} = \frac{v_1 + v_2}{2} \quad \text{Eq. 2-2} \]

Similarly, let \( v_{do} \) and \( v_{cmo} \) denote the difference and common-mode signals at the op-amp input. We have

\[ v_{do} = v_p - v_n \quad \text{and} \quad v_{cmo} = \frac{v_p + v_n}{2} \quad \text{Eq. 2-3} \]

It is clear from the circuit diagram that

\[ v_p = \frac{R_2}{R_1 + R_2} v_2 \quad \text{Eq. 2-4} \]
\[ v_o = v_o + \frac{R_2}{R_1 + R_2} (v_1 - v_o) = v_o \frac{R_1}{R_1 + R_2} + v_1 \frac{R_2}{R_1 + R_2} \]  
Eq. 2-5

The output signal \( v_o \) expressed in terms of the op-amp differential gain \( A_d \) and common-mode gain \( A_{cm} \) is

\[ v_o = A_d v_{do} + A_{cm} v_{cmo} \]  
Eq. 2-6

Using Eqs. 2-3, 2-4, and 2-5 in Eq. 2-6 yields the following expression for \( v_o \):

\[ v_o = \frac{A_d}{R_1 + R_2} (R_2 v_d - v_o R_1) + \frac{A_{cm}}{R_1 + R_2} (R_2 v_{cm} + v_o R_1 / 2) \]  
Eq. 2-7

Collecting all the terms containing \( v_o \) we obtain the following expression for \( v_o \) (in terms of \( v_d \) and \( v_{cm} \) of Eq. 2-2):

\[ v_o = \frac{A_d}{R_1 + R_2} \frac{R_2}{R_1} v_d + \frac{A_{cm}}{R_1 + R_2} \frac{R_2}{R_1} v_{cm} \]  
Eq. 2-8

The denominator of the above equation is approximately equal to \( A_d \) since \( A_d \gg A_{cm} \) and \( A_d \gg R_2/R_1 \) (the gain of the ideal difference amplifier). We thus have

\[ v_o \approx \frac{R_2}{R_1} v_d + \frac{1}{CMRR_o} \frac{R_2}{R_1} v_{cm} \]  
Eq. 2-9

where \( CMRR_o = A_d / A_{cm} \) is the common-mode rejection ratio of the op-amp. It is clear from the above equation that the differential voltage gain \( A_{d-amp} \) and common-mode gain \( A_{cm-amp} \) of the amplifier are as described below:

\[ A_{d-amp} = \frac{R_2}{R_1} \quad A_{cm-amp} = \frac{1}{CMRR_o} \frac{R_2}{R_1} \]  
Eq. 2-10

The common-mode rejection ratio of the difference amplifier is the same as that of the op-amp, as demonstrated below:

\[ CMRR_{amp} = A_{d-amp} / A_{cm-amp} = CMRR_o \]  
Eq. 2-11

As mentioned earlier, the common mode signal can be orders of magnitude higher than the difference signal: the common-mode component of Eq. 2-9 can thus be significant. It is clear from the above discussion that the basic difference amplifier suffers from two disadvantages: low input resistance, and inadequate common-mode rejection.

The instrumentation amplifier

The instrumentation amplifier depicted in Fig. 2-3 does not suffer from the disadvantages listed above; it has high input resistance and high CMRR. It is clear from the circuit diagram that the input resistance seen by the source is governed by the input resistance of the op-amps used in the circuit. The input resistance of the instrumentation amplifier is thus very high.
The instrumentation amplifier consists of an input stage followed by a second stage (which is just a basic difference amplifier). It is easily shown that the differential voltage gain of the first stage is \((1 + 2R_2/R_1)\). We know that the differential gain of the second stage is \(R_4/R_3\). The overall differential gain of the instrumentation amplifier is thus

\[
\frac{V_o}{V_0} = \left(1 + \frac{2R_2}{R_1}\right) \frac{R_4}{R_3} \quad \text{Eq. 2-12}
\]

It is easily shown that the common-mode voltage gain of the first stage is unity. We know (see Eq. 2-10) that the common mode gain of the second stage is \(R_4/(R_3 \cdot \text{CMRR}_o)\). The overall common-mode gain of the instrumentation amplifier is thus

\[
A_{cm-amp} = \frac{1}{\text{CMRR}_o} \frac{R_4}{R_3} \quad \text{Eq. 2-13}
\]

The ratio of Eqs. 2-12 and 2-13 gives the CMRR of the instrumentation amplifier. We have

\[
\text{CMRR}_{amp} = \left(1 + \frac{2R_2}{R_1}\right) \frac{\text{CMRR}_o}{R_3} \quad \text{Eq. 2-14}
\]

The CMRR of the instrumentation amplifier is thus greater than that of the op-amps by a factor \((1 + 2R_2/R_1)\) which can be large. In fact, if we set \(R_4 = R_3\), we see from Eq. 2-12 that this multiplying factor is the (large) differential voltage gain of the instrumentation amplifier.

Figure 2-3: An instrumentation amplifier
PRELAB

1. The typical voltage gain $A_d$ of an LM 741 op-amp is 200,000 and the typical CMRR is 90dB. Use these values to calculate the common mode gain $A_{cm}$ of the 741 op-amp.
2. Prove that the differential voltage gain of the input stage of the instrumentation amplifier is $(1 + 2R_2/R_1)$.
3. Prove that the common-mode voltage gain of the input stage of the instrumentation amplifier is unity.
4. Design an instrumentation amplifier that has a differential voltage gain of 50.
5. Simulate your instrumentation amplifier using PSpice (use uA 741 op-amps) and measure the following parameters. Also compare your measurements with values predicted by theory.
   a) Differential voltage gain of the instrumentation amplifier
   b) Common-mode voltage gain of the instrumentation amplifier
   c) Input resistance of the instrumentation amplifier

IN LAB

1. Use the circuit of Fig. 2-4 to provide the differential input to the instrumentation amplifier. What is the theoretical value of the differential voltage $v_d = v_1 - v_2$? Measure the differential input voltage. Why can't you view the differential input signal on the oscilloscope using a single scope probe?

![Fig. 2-4: Circuit to generate test input signals](image)

2. Construct the instrumentation amplifier that you designed in the prelab. Test the amplifier using the input signals from the circuit of Fig. 2-4. Measure the differential gain of the amplifier. Compare with the expected amplifier performance.