Rotational Dynamics

**Purpose:** Investigate rotational dynamics

**Equipment:** Rotational Motion Apparatus, String + smart pulley, Computer Science Workshop 750, Weight set/hanger

**Discussion:** Essentially all of the relations between physical quantities associated with linear motion that we saw in the laboratory on Newton’s 2nd Law have analogous counterparts in context of rotational motion. Consider a mass “m” attached to a weightless string “r.” Using Newton’s second law as a starting point and substituting rotational equivalents, the following derivation results.

\[ F = ma \]

Convert linear acceleration to angular acceleration

\[ F = m\alpha r \]

Multiply both sides by “r”

\[ rF = m\alpha r \]

Rearrange and substitute \( \tau = rF \)

\[ \tau = mr^2\alpha \]

the term “mr^2” is being accelerated by the torque. This is the **moment of inertia** “I” with units of kg\(\cdot\)m^2

\[ \tau = I\alpha \]

consider the dynamics of the set up used in today’s lab

\[ I = \text{moment of inertia of rotating bar} \]

\[ r = \text{radius of this wheel} \]

\[ \tau = I\alpha \]

\[ \tau = rT \]

\[ F_{\text{net}} = \Sigma F \]

\[ ma = mg - T \]

\[ T = mg - ma \]

**Moment of inertia** is a body’s resistance to changes in rotational motion. It depends not only on the mass, but also the distribution of the mass. Consider a merry-go-round. Obviously more riders will require more effort to accelerate the system. Now consider the riders distribution. When the riders are near the center, the merry-go-round is easier to
accelerate than when the riders are near the edge. One reasonable explanation involves the distance traveled by the riders in equal times. Those near the edge must travel further in the same amount of time and would require more force to accelerate at the same rate as those near the center. The moment of inertia or resistance to changes in rotation is greater when the riders are near the edge even though the mass of the merry-go-round remains constant.

**Procedure A. moment of inertia of a rotating bar**

1. Be sure the platform is level. (Your instructor will show you how to do this.) After leveling, begin with both stops centered at the 24-cm and nothing else on the platform.

2. Beginning with a driving mass of 100 grams on the end of the hanging string. That’s 50g for the hanger plus an additional 50g mass. Wind the bar so that the hanger is just below the pulley.

3. Start the interface and release the bar. Stop the interface once the driving mass reaches the bottom.

4. Record the slope of the v-t graph from the interface. This is the linear acceleration of the hanging mass and must be converted to angular acceleration, \( \alpha \).

5. Repeat this process 4 more times, each time increasing the hanging mass by 50 grams.

6. Calculate torque and angular acceleration values for each run. **Plot torque vs. angular acceleration.** The slope of the trendline is your experimental value for the moment of inertia of the rotating bar. The actual value is 0.0126 kg\(\cdot\)m\(^2\). Data table for Part A should look like the one below. **Moment of inertia and % error are the results for Part A.**

<table>
<thead>
<tr>
<th>trial</th>
<th>m(kg)</th>
<th>a(m/s(^2))</th>
<th>T(N)</th>
<th>(\alpha)(rad/s(^2))</th>
<th>(\tau)(m\cdot N))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.100</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>.150</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>.200</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>.250</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>.300</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*notes: \( \alpha = \frac{a}{r} \) “r” is the radius of the wheel that the string is wrapped around.

\( m = \text{driving mass (the mass hanging on the string)} \)

\( T = mg - ma \ (g = 9.81 \text{m/s}^2) \)

\( \tau = rT \ (r= \text{radius of driving wheel, T as above.}) \)
Part B: moment of inertia of a complex object

**Purpose:** to show that the moment of inertia of a complex object is the sum of the moments of the parts.

**Discussion:** The moment of inertia of the experimental system is the sum of the moments of the bar from procedure A and point masses added at a distance “r” from the center of rotation. Theory holds that the moment of inertia of a point mass is given by $I = mr^2$ for one mass or $I = 2mr^2$ for the two masses used in this procedure.

**Procedure:**

1. Place a 275g mass at the 20.0 cm mark on each side of the rotating platform.

2. Repeat the procedure as in part A to determine the moment of inertia of the bar with masses combination.

3. Subtract the moment of inertia of the bar found in part A to find the moment of inertia of the point masses. Do a relative error calculation using this value as the experimental and the calculated: $I=2mr^2$ as the actual. “m” is 275g and “r” is 20.0 cm. Discuss errors greater than 5%. This is result B

**Supplemental Questions: Please write the questions and show all work.**

1. Which would have a greater moment of inertia, a thing ring or a solid disk? Both have the same mass and radius. Explain.

2. The moment of inertia of a 4.0m long board pivoted at its center is 25kgm$^2$. Find the moment of inertia when it is used as a see-saw with two 50kg kids sitting 1.8mm from the pivot.

3. Weights are sometimes added to the arms of a boomerang. What effect would this have on the spinning of the boomerang? Where should they be placed for greatest effect? Please indicate the location and explain your reasoning.